

# General Relativity

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21-Oct-1998

## 1 Introduction

## 2 The field equations of general relativity

### 2.1 Full $R_{ij}$

$$\begin{aligned} R_{00} = & -\frac{1}{2}g^{11}\frac{\partial^2 g_{11}}{\partial x^0\partial x^0} + g^{11}\frac{\partial^2 g_{01}}{\partial x^0\partial x^1} - \frac{1}{2}g^{11}\frac{\partial^2 g_{00}}{\partial x^1\partial x^1} - g^{12}\frac{\partial^2 g_{12}}{\partial x^0\partial x^0} \\ & + g^{12}\frac{\partial^2 g_{02}}{\partial x^0\partial x^1} + g^{12}\frac{\partial^2 g_{01}}{\partial x^0\partial x^2} - g^{12}\frac{\partial^2 g_{00}}{\partial x^1\partial x^2} - g^{13}\frac{\partial^2 g_{13}}{\partial x^0\partial x^0} \\ & + g^{13}\frac{\partial^2 g_{01}}{\partial x^0\partial x^3} - g^{13}\frac{\partial^2 g_{00}}{\partial x^1\partial x^3} - \frac{1}{2}g^{22}\frac{\partial^2 g_{22}}{\partial x^0\partial x^0} + g^{22}\frac{\partial^2 g_{02}}{\partial x^0\partial x^2} \\ & - \frac{1}{2}g^{22}\frac{\partial^2 g_{00}}{\partial x^2\partial x^2} - g^{23}\frac{\partial^2 g_{23}}{\partial x^0\partial x^0} + g^{23}\frac{\partial^2 g_{03}}{\partial x^0\partial x^2} + g^{23}\frac{\partial^2 g_{02}}{\partial x^0\partial x^3} \\ & - g^{23}\frac{\partial^2 g_{00}}{\partial x^2\partial x^3} - \frac{1}{2}g^{33}\frac{\partial^2 g_{33}}{\partial x^0\partial x^0} + g^{33}\frac{\partial^2 g_{03}}{\partial x^0\partial x^3} - \frac{1}{2}g^{33}\frac{\partial^2 g_{00}}{\partial x^3\partial x^3} \\ & + \frac{1}{4}g^{00}g^{11}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{11}}{\partial x^0} - \frac{1}{2}g^{00}g^{11}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{01}}{\partial x^1} + \frac{1}{2}g^{00}g^{12}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{12}}{\partial x^0} - \frac{1}{2}g^{00}g^{12}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{02}}{\partial x^1} \\ & - \frac{1}{2}g^{00}g^{12}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{01}}{\partial x^2} + \frac{1}{2}g^{00}g^{12}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{00}}{\partial x^2} - \frac{1}{4}g^{01}g^{11}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{11}}{\partial x^1} + \frac{1}{2}g^{01}g^{11}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{11}}{\partial x^0} \\ & - g^{01}g^{11}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{01}}{\partial x^1} + \frac{1}{4}g^{01}g^{11}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{11}}{\partial x^0} + \frac{1}{2}g^{01}g^{11}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{01}}{\partial x^1} + \frac{1}{2}g^{00}g^{13}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{13}}{\partial x^0} \\ & - \frac{1}{2}g^{00}g^{13}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{03}}{\partial x^1} - \frac{1}{2}g^{00}g^{13}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{01}}{\partial x^3} + \frac{1}{2}g^{00}g^{13}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{00}}{\partial x^3} + \frac{1}{4}g^{00}g^{22}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{22}}{\partial x^0} \\ & - \frac{1}{2}g^{00}g^{22}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{02}}{\partial x^2} + g^{01}g^{03}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{00}}{\partial x^3} - g^{01}g^{03}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{00}}{\partial x^3} - \frac{1}{2}g^{01}g^{12}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{11}}{\partial x^2} \\ & + g^{01}g^{12}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{12}}{\partial x^0} - g^{01}g^{12}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{02}}{\partial x^1} - g^{01}g^{12}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{01}}{\partial x^2} + g^{01}g^{12}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{01}}{\partial x^2} \\ & + \frac{1}{2}g^{01}g^{12}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{00}}{\partial x^2} - \frac{1}{2}g^{02}g^{11}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{12}}{\partial x^1} + \frac{1}{4}g^{02}g^{11}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{11}}{\partial x^2} + g^{02}g^{11}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{01}}{\partial x^2} \\ & - g^{02}g^{11}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{01}}{\partial x^2} + \frac{1}{2}g^{02}g^{11}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{12}}{\partial x^0} + \frac{1}{2}g^{02}g^{11}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{02}}{\partial x^1} - \frac{1}{2}g^{02}g^{11}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{01}}{\partial x^2} \\ & + \frac{1}{2}g^{02}g^{11}\frac{\partial g_{02}}{\partial x^0}\frac{\partial g_{11}}{\partial x^0} - g^{02}g^{11}\frac{\partial g_{02}}{\partial x^0}\frac{\partial g_{01}}{\partial x^1} - \frac{1}{4}g^{02}g^{11}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{00}}{\partial x^2} + \frac{1}{2}g^{02}g^{11}\frac{\partial g_{01}}{\partial x^1}\frac{\partial g_{00}}{\partial x^2} \\ & + \frac{1}{2}g^{00}g^{23}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{23}}{\partial x^0} - \frac{1}{2}g^{00}g^{23}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{03}}{\partial x^2} - \frac{1}{2}g^{00}g^{23}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{02}}{\partial x^3} + \frac{1}{2}g^{00}g^{23}\frac{\partial g_{00}}{\partial x^2}\frac{\partial g_{00}}{\partial x^3} \\ & - \frac{1}{2}g^{01}g^{13}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{11}}{\partial x^3} + g^{01}g^{13}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{13}}{\partial x^0} - g^{01}g^{13}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{03}}{\partial x^1} - g^{01}g^{13}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{01}}{\partial x^3} \\ & + g^{01}g^{13}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{01}}{\partial x^3} + g^{01}g^{13}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{00}}{\partial x^3} - \frac{1}{2}g^{01}g^{13}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{00}}{\partial x^3} + \frac{1}{4}g^{01}g^{22}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{22}}{\partial x^1} \\ & - \frac{1}{2}g^{01}g^{22}\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{12}}{\partial x^2} + \frac{1}{2}g^{01}g^{22}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{22}}{\partial x^0} - g^{01}g^{22}\frac{\partial g_{01}}{\partial x^0}\frac{\partial g_{02}}{\partial x^2} - \frac{1}{4}g^{01}g^{22}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{22}}{\partial x^0} \\ & + \frac{1}{2}g^{01}g^{22}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{02}}{\partial x^2} + g^{01}g^{22}\frac{\partial g_{02}}{\partial x^0}\frac{\partial g_{02}}{\partial x^1} - g^{01}g^{22}\frac{\partial g_{02}}{\partial x^0}\frac{\partial g_{02}}{\partial x^1} + \frac{1}{2}g^{01}g^{22}\frac{\partial g_{00}}{\partial x^2}\frac{\partial g_{12}}{\partial x^0} \end{aligned}$$





























































































































$$\begin{aligned}
& -\frac{1}{2}g^{00}g^{33}\frac{\partial g_{33}}{\partial x^0}\frac{\partial g_{03}}{\partial x^3} - \frac{1}{4}g^{11}g^{22}\frac{\partial g_{11}}{\partial x^2}\frac{\partial g_{33}}{\partial x^2} + \frac{1}{2}g^{11}g^{22}\frac{\partial g_{11}}{\partial x^2}\frac{\partial g_{23}}{\partial x^3} - \frac{1}{4}g^{11}g^{22}\frac{\partial g_{22}}{\partial x^1}\frac{\partial g_{33}}{\partial x^1} \\
& + \frac{1}{2}g^{11}g^{22}\frac{\partial g_{22}}{\partial x^1}\frac{\partial g_{13}}{\partial x^3} - g^{11}g^{22}\frac{\partial g_{23}}{\partial x^1}\frac{\partial g_{13}}{\partial x^2} + \frac{1}{4}g^{11}g^{33}\frac{\partial g_{11}}{\partial x^3}\frac{\partial g_{33}}{\partial x^3} - \frac{1}{2}g^{11}g^{33}\frac{\partial g_{33}}{\partial x^1}\frac{\partial g_{13}}{\partial x^3} \\
& + \frac{1}{4}g^{22}g^{33}\frac{\partial g_{22}}{\partial x^3}\frac{\partial g_{33}}{\partial x^3} - \frac{1}{2}g^{22}g^{33}\frac{\partial g_{33}}{\partial x^2}\frac{\partial g_{23}}{\partial x^3} + \frac{1}{2}g^{00}g^{11}\left(\frac{\partial g_{13}}{\partial x^0}\right)^2 + \frac{1}{2}g^{00}g^{11}\left(\frac{\partial g_{03}}{\partial x^1}\right)^2 \\
& - \frac{1}{2}g^{00}g^{11}\left(\frac{\partial g_{01}}{\partial x^3}\right)^2 + \frac{1}{2}g^{00}g^{22}\left(\frac{\partial g_{23}}{\partial x^0}\right)^2 + \frac{1}{2}g^{00}g^{22}\left(\frac{\partial g_{03}}{\partial x^2}\right)^2 - \frac{1}{2}g^{00}g^{22}\left(\frac{\partial g_{02}}{\partial x^3}\right)^2 \\
& + \frac{1}{4}g^{00}g^{33}\left(\frac{\partial g_{33}}{\partial x^0}\right)^2 + \frac{1}{2}g^{11}g^{22}\left(\frac{\partial g_{23}}{\partial x^1}\right)^2 + \frac{1}{2}g^{11}g^{22}\left(\frac{\partial g_{13}}{\partial x^2}\right)^2 - \frac{1}{2}g^{11}g^{22}\left(\frac{\partial g_{12}}{\partial x^3}\right)^2 \\
& + \frac{1}{4}g^{11}g^{33}\left(\frac{\partial g_{33}}{\partial x^1}\right)^2 + \frac{1}{4}g^{22}g^{33}\left(\frac{\partial g_{33}}{\partial x^2}\right)^2 + \frac{1}{4}(g^{00})^2\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{33}}{\partial x^0} - \frac{1}{2}(g^{00})^2\frac{\partial g_{00}}{\partial x^0}\frac{\partial g_{03}}{\partial x^3} \\
& + \frac{1}{4}(g^{11})^2\frac{\partial g_{11}}{\partial x^1}\frac{\partial g_{33}}{\partial x^1} - \frac{1}{2}(g^{11})^2\frac{\partial g_{11}}{\partial x^1}\frac{\partial g_{13}}{\partial x^3} + \frac{1}{4}(g^{22})^2\frac{\partial g_{22}}{\partial x^2}\frac{\partial g_{33}}{\partial x^2} - \frac{1}{2}(g^{22})^2\frac{\partial g_{22}}{\partial x^2}\frac{\partial g_{23}}{\partial x^3} \\
& + \frac{1}{4}(g^{00})^2\left(\frac{\partial g_{00}}{\partial x^3}\right)^2 + \frac{1}{4}(g^{11})^2\left(\frac{\partial g_{11}}{\partial x^3}\right)^2 + \frac{1}{4}(g^{22})^2\left(\frac{\partial g_{22}}{\partial x^3}\right)^2
\end{aligned}$$

### 3 The field equations of general relativity

Caution !  $g_{i,j} = 0, i \neq j$  and  $\frac{\partial g_{0k}}{\partial x^0} = 0, k \neq 0$

#### 3.1 $R$

$$\begin{aligned}
R = & -g^{00}g^{11}\frac{\partial^2 g_{11}}{\partial x^0\partial x^0} + 2g^{00}g^{11}\frac{\partial^2 g_{01}}{\partial x^0\partial x^1} - g^{00}g^{11}\frac{\partial^2 g_{00}}{\partial x^1\partial x^1} \\
& -g^{00}g^{22}\frac{\partial^2 g_{22}}{\partial x^0\partial x^0} + 2g^{00}g^{22}\frac{\partial^2 g_{02}}{\partial x^0\partial x^2} - g^{00}g^{22}\frac{\partial^2 g_{00}}{\partial x^2\partial x^2} \\
& -g^{00}g^{33}\frac{\partial^2 g_{33}}{\partial x^0\partial x^0} + 2g^{00}g^{33}\frac{\partial^2 g_{03}}{\partial x^0\partial x^3} - g^{00}g^{33}\frac{\partial^2 g_{00}}{\partial x^3\partial x^3} \\
& -g^{11}g^{22}\frac{\partial^2 g_{22}}{\partial x^1\partial x^1} + 2g^{11}g^{22}\frac{\partial^2 g_{12}}{\partial x^1\partial x^2} - g^{11}g^{22}\frac{\partial^2 g_{11}}{\partial x^2\partial x^2} \\
& -g^{11}g^{33}\frac{\partial^2 g_{33}}{\partial x^1\partial x^1} + 2g^{11}g^{33}\frac{\partial^2 g_{13}}{\partial x^1\partial x^3} - g^{11}g^{33}\frac{\partial^2 g_{11}}{\partial x^3\partial x^3} \\
& -g^{22}g^{33}\frac{\partial^2 g_{33}}{\partial x^2\partial x^2} + 2g^{22}g^{33}\frac{\partial^2 g_{23}}{\partial x^2\partial x^3} - g^{22}g^{33}\frac{\partial^2 g_{22}}{\partial x^3\partial x^3} \\
& -\frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{22}}{\partial x^1} + \frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{12}}{\partial x^2} - \frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{22}}{\partial x^0} \\
& + \frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{02}}{\partial x^2} + \frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{01}}{\partial x^1}\frac{\partial g_{22}}{\partial x^0} + \frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{00}}{\partial x^2}\frac{\partial g_{12}}{\partial x^1} \\
& -\frac{1}{2}g^{00}g^{11}g^{22}\frac{\partial g_{00}}{\partial x^2}\frac{\partial g_{11}}{\partial x^2} - g^{00}g^{11}g^{22}\frac{\partial g_{12}}{\partial x^0}\frac{\partial g_{02}}{\partial x^1} - g^{00}g^{11}g^{22}\frac{\partial g_{12}}{\partial x^0}\frac{\partial g_{01}}{\partial x^2} \\
& -g^{00}g^{11}g^{22}\frac{\partial g_{02}}{\partial x^1}\frac{\partial g_{01}}{\partial x^2} - \frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{33}}{\partial x^1} + \frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{00}}{\partial x^1}\frac{\partial g_{13}}{\partial x^3} \\
& -\frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{33}}{\partial x^0} + \frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{11}}{\partial x^0}\frac{\partial g_{03}}{\partial x^3} + \frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{01}}{\partial x^1}\frac{\partial g_{33}}{\partial x^0} \\
& + \frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{00}}{\partial x^3}\frac{\partial g_{13}}{\partial x^1} - \frac{1}{2}g^{00}g^{11}g^{33}\frac{\partial g_{00}}{\partial x^3}\frac{\partial g_{11}}{\partial x^3} - g^{00}g^{11}g^{33}\frac{\partial g_{13}}{\partial x^0}\frac{\partial g_{03}}{\partial x^1} \\
& -g^{00}g^{11}g^{33}\frac{\partial g_{13}}{\partial x^0}\frac{\partial g_{01}}{\partial x^3} - g^{00}g^{11}g^{33}\frac{\partial g_{03}}{\partial x^1}\frac{\partial g_{01}}{\partial x^3} - \frac{1}{2}g^{00}g^{22}g^{33}\frac{\partial g_{00}}{\partial x^2}\frac{\partial g_{33}}{\partial x^2} \\
& + \frac{1}{2}g^{00}g^{22}g^{33}\frac{\partial g_{00}}{\partial x^2}\frac{\partial g_{23}}{\partial x^3} - \frac{1}{2}g^{00}g^{22}g^{33}\frac{\partial g_{00}}{\partial x^3}\frac{\partial g_{22}}{\partial x^3} - \frac{1}{2}g^{00}g^{22}g^{33}\frac{\partial g_{22}}{\partial x^0}\frac{\partial g_{33}}{\partial x^0} \\
& + \frac{1}{2}g^{00}g^{22}g^{33}\frac{\partial g_{22}}{\partial x^0}\frac{\partial g_{03}}{\partial x^3} + \frac{1}{2}g^{00}g^{22}g^{33}\frac{\partial g_{02}}{\partial x^2}\frac{\partial g_{33}}{\partial x^0} - g^{00}g^{22}g^{33}\frac{\partial g_{23}}{\partial x^0}\frac{\partial g_{03}}{\partial x^2}
\end{aligned}$$



















$$\begin{aligned}
& + \frac{1}{2} g^{00} g^{11} (g^{33})^{-1} \frac{\partial^2 g_{11}}{\partial x^0 \partial x^0} - g^{00} g^{11} (g^{33})^{-1} \frac{\partial^2 g_{01}}{\partial x^0 \partial x^1} + \frac{1}{2} g^{00} g^{11} (g^{33})^{-1} \frac{\partial^2 g_{00}}{\partial x^1 \partial x^1} \\
& + \frac{1}{2} g^{00} g^{22} (g^{33})^{-1} \frac{\partial^2 g_{22}}{\partial x^0 \partial x^0} - g^{00} g^{22} (g^{33})^{-1} \frac{\partial^2 g_{02}}{\partial x^0 \partial x^2} + \frac{1}{2} g^{00} g^{22} (g^{33})^{-1} \frac{\partial^2 g_{00}}{\partial x^2 \partial x^2} \\
& + \frac{1}{2} g^{11} g^{22} (g^{33})^{-1} \frac{\partial^2 g_{22}}{\partial x^1 \partial x^1} - g^{11} g^{22} (g^{33})^{-1} \frac{\partial^2 g_{12}}{\partial x^1 \partial x^2} + \frac{1}{2} g^{11} g^{22} (g^{33})^{-1} \frac{\partial^2 g_{11}}{\partial x^2 \partial x^2} \\
& + \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^1} \frac{\partial g_{22}}{\partial x^1} - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^1} \frac{\partial g_{12}}{\partial x^2} + \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{11}}{\partial x^0} \frac{\partial g_{22}}{\partial x^0} \\
& - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{11}}{\partial x^0} \frac{\partial g_{02}}{\partial x^2} - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{01}}{\partial x^1} \frac{\partial g_{22}}{\partial x^0} - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^2} \frac{\partial g_{12}}{\partial x^1} \\
& + \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^2} \frac{\partial g_{11}}{\partial x^2} + \frac{1}{2} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{12}}{\partial x^0} \frac{\partial g_{02}}{\partial x^1} + \frac{1}{2} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{12}}{\partial x^0} \frac{\partial g_{01}}{\partial x^2} \\
& + \frac{1}{2} g^{00} g^{11} g^{22} (g^{33})^{-1} \frac{\partial g_{02}}{\partial x^1} \frac{\partial g_{01}}{\partial x^2} - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \left( \frac{\partial g_{12}}{\partial x^0} \right)^2 - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \left( \frac{\partial g_{02}}{\partial x^1} \right)^2 \\
& - \frac{1}{4} g^{00} g^{11} g^{22} (g^{33})^{-1} \left( \frac{\partial g_{01}}{\partial x^2} \right)^2 - \frac{1}{4} (g^{00})^2 g^{11} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^0} \frac{\partial g_{11}}{\partial x^0} + \frac{1}{4} (g^{00})^2 g^{11} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^0} \frac{\partial g_{01}}{\partial x^1} \\
& - \frac{1}{4} g^{00} (g^{11})^2 (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^1} \frac{\partial g_{11}}{\partial x^1} + \frac{1}{4} g^{00} (g^{11})^2 (g^{33})^{-1} \frac{\partial g_{11}}{\partial x^0} \frac{\partial g_{01}}{\partial x^1} - \frac{1}{4} (g^{00})^2 g^{22} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^0} \frac{\partial g_{22}}{\partial x^0} \\
& + \frac{1}{4} (g^{00})^2 g^{22} (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^0} \frac{\partial g_{02}}{\partial x^2} - \frac{1}{4} g^{00} (g^{22})^2 (g^{33})^{-1} \frac{\partial g_{00}}{\partial x^2} \frac{\partial g_{22}}{\partial x^2} + \frac{1}{4} g^{00} (g^{22})^2 (g^{33})^{-1} \frac{\partial g_{22}}{\partial x^0} \frac{\partial g_{02}}{\partial x^2} \\
& - \frac{1}{4} (g^{11})^2 g^{22} (g^{33})^{-1} \frac{\partial g_{11}}{\partial x^1} \frac{\partial g_{22}}{\partial x^1} + \frac{1}{4} (g^{11})^2 g^{22} (g^{33})^{-1} \frac{\partial g_{11}}{\partial x^1} \frac{\partial g_{12}}{\partial x^2} + \frac{1}{4} (g^{11})^2 g^{22} (g^{33})^{-1} \frac{\partial g_{12}}{\partial x^1} \frac{\partial g_{11}}{\partial x^2} \\
& + \frac{1}{4} g^{11} (g^{22})^2 (g^{33})^{-1} \frac{\partial g_{12}}{\partial x^1} \frac{\partial g_{22}}{\partial x^2} - \frac{1}{4} g^{11} (g^{22})^2 (g^{33})^{-1} \frac{\partial g_{11}}{\partial x^2} \frac{\partial g_{22}}{\partial x^2} + \frac{1}{4} g^{11} (g^{22})^2 (g^{33})^{-1} \frac{\partial g_{22}}{\partial x^1} \frac{\partial g_{12}}{\partial x^2} \\
& - \frac{1}{4} (g^{00})^2 g^{11} (g^{33})^{-1} \left( \frac{\partial g_{00}}{\partial x^1} \right)^2 - \frac{1}{4} g^{00} (g^{11})^2 (g^{33})^{-1} \left( \frac{\partial g_{11}}{\partial x^0} \right)^2 - \frac{1}{4} (g^{00})^2 g^{22} (g^{33})^{-1} \left( \frac{\partial g_{00}}{\partial x^2} \right)^2 \\
& - \frac{1}{4} g^{00} (g^{22})^2 (g^{33})^{-1} \left( \frac{\partial g_{22}}{\partial x^0} \right)^2 - \frac{1}{4} (g^{11})^2 g^{22} (g^{33})^{-1} \left( \frac{\partial g_{11}}{\partial x^2} \right)^2 - \frac{1}{4} g^{11} (g^{22})^2 (g^{33})^{-1} \left( \frac{\partial g_{22}}{\partial x^1} \right)^2
\end{aligned}$$